NASA Technical Memorandum 4695

# Optimization of Measurements With Pressure Sensitive Paints

*Donald M. Oglesby*
*Old Dominion University • Norfolk, Virginia*

*Chith K. Puram*
*ViGYAN, Inc. • Hampton, Virginia*

*Billy T. Upchurch*
*Langley Research Center • Hampton, Virginia*

June 1995

Available electronically at the following URL address: http://techreports.larc.nasa.gov/ltrs/ltrs.html

Printed copies available from the following:

NASA Center for AeroSpace Information
800 Elkridge Landing Road
Linthicum Heights, MD 21090-2934
(301) 621-0390

National Technical Information Service (NTIS)
5285 Port Royal Road
Springfield, VA 22161-2171
(703) 487-4650

# Abstract

*Use of luminescent paints for the measurement of global pressures on wind tunnel model surfaces requires a full understanding of the inherent accuracy of the technique. Theoretical emission of the paint luminophor follows the well known Stern-Volmer relation. Inherent in this relation are fundamental limits to achievable sensitivity and accuracy. Equations for relative error in pressure as a function of relative signal intensity (emittance), relative error in pressure as a function of pressure, and the relationship between sensitivity and pressure are derived and represented graphically.*

## Introduction

Measurement of surface pressures is very important in fluid mechanics because surface pressures are used in airplane design and in aircraft performance calculations. The conventional technique of measuring pressure by using holes drilled in wind tunnel models makes model fabrication both tedious and expensive. Recently Morris et al. have demonstrated a technique of applying luminescent paint on a model surface and computing surface pressures from the measurements of light emission from the painted surface (refs. 1 and 2). A general account of this technique is given by Crites (ref. 3). The technique is based on the fact that the fluorescence or phosphorescence of many luminophors is quenched by oxygen. As the oxygen partial pressure increases the luminescent intensity of the paint decreases.

It has been found that the luminescence of pressure sensitive paint (PSP) is governed by the well known Stern-Volmer relation (see ref. 3) given by

$$\frac{I_0}{I} = 1 + K_{sv} P_{O_2}$$

For air pressure, the Stern-Volmer relation may be expressed as

$$\frac{I_0}{I} = 1 + KP \tag{1}$$

In using pressure sensitive paint for pressure measurements, it is important to minimize measurement uncertainties by optimizing the measured signal (light intensity) and the sensitivity of the paint. There are limitations inherent in the Stern-Volmer relation that must be considered.

Sajben presents a detailed study of factors that influence accuracy of PSP measurements (ref. 4). He considered the effect on error in PSP measurements of variables such as measurement of luminescent intensity, reference light intensity, surface temperature, and inherent uncertainties associated with the Stern-Volmer relation. His generalized treatment incorporates these measurement uncertainties by using terms designated as influence coefficients $\phi_n$. He then considers the effect of wind tunnel conditions on these influence coefficients. The treatment reported herein is simplified and only the uncertainty inherent in using the Stern-Volmer relation for data reduction with a given uncertainty in the luminescent measurement is considered.

The experimental parameter that directly relates to the uncertainty in measuring pressure is the uncertainty in measuring the light emitted by the paint. For a typical 8-bit, standard grade charge-coupled device (CCD) video camera with a 10-sec integration time, the uncertainty in measuring a constant light source is about 0.5 percent. For a 16-bit scientific grade camera, the uncertainty could be less than 0.1 percent. A 0.5-percent uncertainty of measured emittance *E* was chosen to illustrate the relationships derived in this paper.

## Nomenclature

| | |
|---|---|
| CCD | charge-coupled device |
| $E$ | emittance, $I/I_0$ |
| $I$ | emission intensity at pressure $P$ |
| $I_0$ | emission intensity at zero oxygen partial pressure |
| $I_1$ | emission intensity at some wind tunnel reference pressure (wind off) |
| $I_2$ | emission intensity during wind tunnel run conditions (wind on) |
| $K$ | Stern-Volmer constant for air ($0.209K_{sv}$, the true Stern-Volmer constant) |
| $K_{sv}$ | Stern-Volmer constant |
| $P$ | air pressure (assumes constant partial pressure of oxygen) |
| $P_{O_2}$ | partial pressure of oxygen |
| PSP | pressure sensitive paint |
| $S$ | sensitivity, $dE/dP$ |

| | |
|---|---|
| $\Delta E$ | uncertainty in measured emittance |
| $\Delta P$ | difference in pressure |
| $\varepsilon$ | relative error in pressure, $dP/P$ |
| $\phi_n$ | influence coefficients |

## Dependence of Relative Error in Pressure on Signal Intensity

A wind tunnel pressure value is determined by measuring the intensity of light emitted from the PSP on the model surface. It is obvious that the relative error in determining the pressure will be greater for very weak signals or very intense signals. However, the exact relationship between signal intensity and relative error in the resulting pressure value is inherent in the Stern-Volmer relation.

In a manner similar to the definition of transmittance, emittance $E$ is defined as

$$E = \frac{I}{I_0} \tag{2}$$

In terms of $E$, equation (1) can be written as

$$E = (1 + KP)^{-1} \tag{3}$$

Differentiating $E$ with respect to $P$,

$$\frac{dE}{dP} = -(1 + KP)^{-2} K \tag{4}$$

Because the relative error in a pressure value caused by an error in measuring emittance is a concern, equation (4) can be divided by $P$ to give

$$\frac{dP}{P} = -\frac{(1 + KP)^2}{KP} dE \tag{5}$$

For small finite errors in $E$ and $P$, equation (5) can be written as

$$\frac{\Delta P}{P} = \frac{(1 + KP)^2}{KP} \Delta E \tag{6}$$

Substituting equation (3) in equation (6)

$$\frac{\Delta P}{P} = \frac{\Delta E}{(1 - E)E} \tag{7}$$

As can be seen from equation (7), the relative error in pressure is a function of $E$ and is independent of $K$. Figure 1 shows the variation of relative error in pressure $P/P$ as a function of emittance for a 0.5-percent error in intensity measurement. The lowest relative error occurs at an emittance of 0.5. However, relatively small errors occur in the emittance range of 0.2 to 0.8.

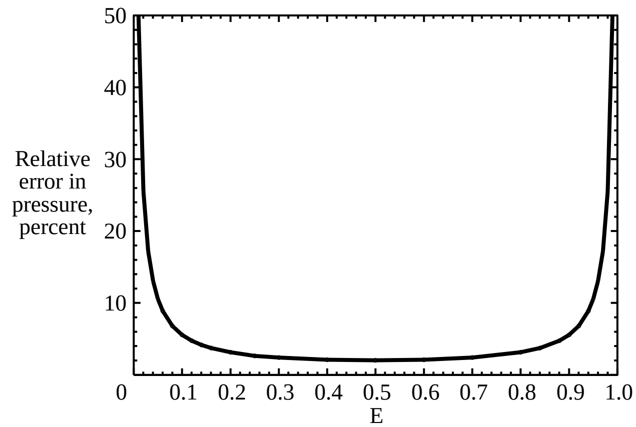


Figure 1. Relative error in pressure as a function of emittance for constant error in emittance of 0.5 percent.

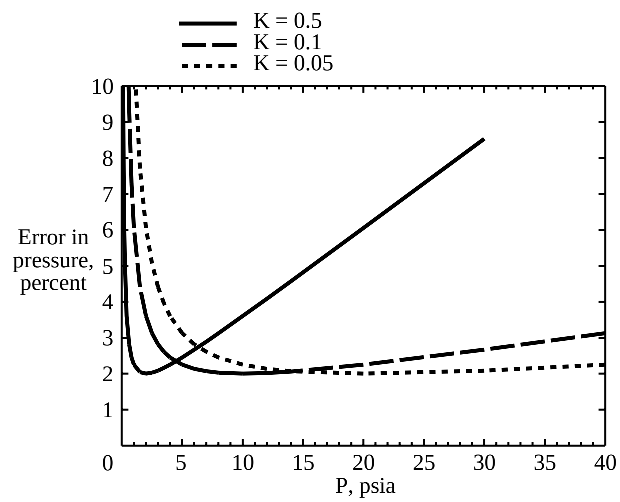


Figure 2. Relative error in pressure as a function of pressure for a constant error in emittance of 0.5 percent.

## Error in Pressure as a Function of Pressure and *K*

For a particular paint and a given uncertainty in measuring emittance, the uncertainty in the value of pressure is not the same at all pressures. Although there are numerous experimental factors that can lead to errors in pressure measurement, it is important to understand that there are also intrinsic limits inherent in the Stern-Volmer relation.

Equation (6) gives the relative error in pressure measurement as a function of pressure for a fixed error, $\Delta E$. Figure 2 shows plots of relative error in pressure as a function of pressure for several values of $K$. ($\Delta E = 0.005$ is used for all plots.)

Ideally, the error in pressure measurement should be minimized. Let relative error in pressure measurement ε be defined by

$$\varepsilon = -\frac{dP}{P} \tag{8}$$

Substituting equation (8) into equation (5) and differentiating it with respect to $P$, for small finite changes in $E$, we get

$$\frac{d\varepsilon}{dP} = \frac{(K^2P^2 - 1)}{P^2}\frac{\Delta E}{K} \tag{9}$$

Error ε will be minimum when its derivative is zero. Equation (9) will be zero when

$$KP = 1$$

or error will be least when

$$P = \frac{1}{K} \tag{10}$$

From this equation we see that for a given value of $K$, there is only one pressure at which the relative error in pressure will be minimum. By substituting equation (10) into equation (6) and simplifying, it is apparent that the relative error at the pressure of minimum error is

$$\frac{\Delta P}{P} = 4\Delta E \tag{11}$$

Notice in figure 2 that error increases rapidly at pressures below the pressure of minimum error, but increases much more slowly at higher pressures. This pattern of increase means that the working range for a given paint (a given value of $K$) extends upward from the pressure of minimum error. For example, if air pressure measurements are made at atmospheric pressure or above, $K$ (determined in air) should be about 0.07 psia$^{-1}$.

## Sensitivity Analysis

Without considering other factors, one might look at figure 2 and be inclined to conclude that a small value of $K$ gives the lowest relative error over the broadest pressure range and would therefore be desirable. However, typical emittance response curves for different values of $K$, shown in figure 3, show that for measurements at high pressures, better sensitivity can be achieved with larger values of $K$. To measure small variations in pressure, the change in luminescence in the paint should be high for a small pressure change $P$ or $dE/dP$ should be large.

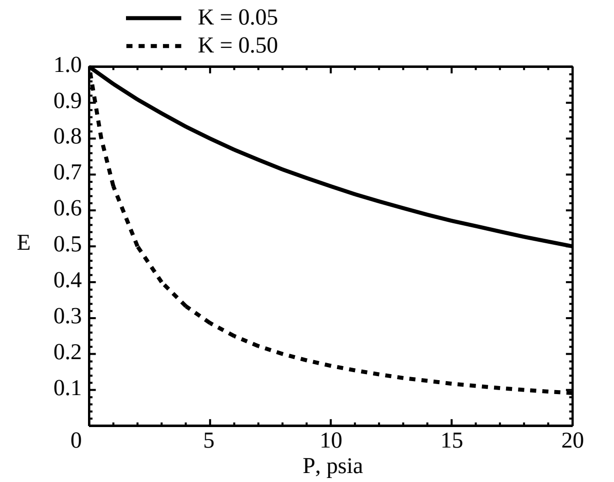


Figure 3. Emittance as a function of pressure for two values of $K$.

Sensitivity of the paint $S$ can be defined as the ratio of change in output signal or emittance for a small change in pressure, or

$$S = -\frac{dE}{dP} \tag{12}$$

With this definition equation (4) can be written as

$$S = \frac{K}{(1 + KP)^2} \tag{13}$$

This equation can be written as an equation in $K$ as

$$(1 + KP)^2 S - K = 0 \tag{14}$$

For the solution of this equation for $K$ to be real

$$0 \le PS \le 0.25 \tag{15}$$

Figure 4 shows the variation of sensitivity of the paint $S$ with pressure for three values of $K$.

A logarithmic scale is used in figure 4 to more clearly show the relationship between $S$, $K$, and $P$ over a wider range of pressure. From equation (13) we may see that at zero pressure $S = K$, which represents the limit of sensitivity for a given paint. Thus for any value of $K$, the greatest sensitivity is attained at lower pressures. However, as we have shown, this greatest sensitivity at lower pressures does not necessarily give the lowest relative error in the pressure value. The optimum working pressure range for a given value of $K$ is on each side of the point where the plot of $S$ and $P$ touches the limiting line of $PS = 0.25$. The pressure at which the sensitivity curve touches the boundary line corresponds to the pressure of minimum relative error in pressure.

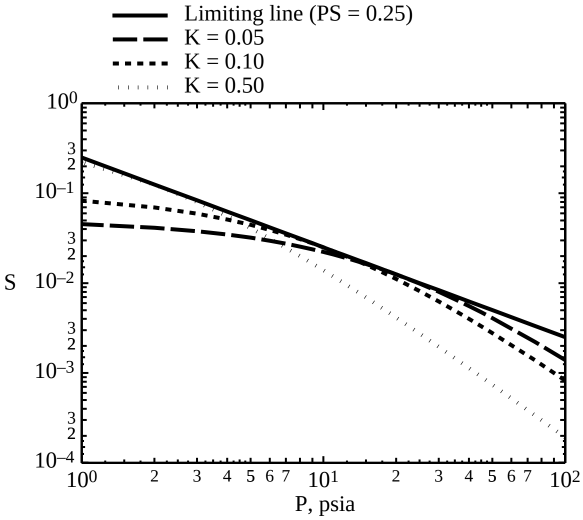


Figure 4. Sensitivity as a function of pressure.

## Stern-Volmer Relation in Wind Tunnel Measurements

The $I_0$ in the Stern-Volmer relation represents the intensity of emission at oxygen partial pressure of zero. However, this pressure is not always practical in wind tunnel measurements. Instead of trying to achieve an oxygen partial pressure of zero, the intensity of emission at wind off $I_1$ is used and the pressure at wind off is considered the reference pressure $P_1$. In terms of the Stern-Volmer relation for air, this ratio takes the form of the Stern-Volmer relation for two pressures.

Wind off $$\frac{I_0}{I_1} = 1 + KP_1 \tag{16}$$

Wind on $$\frac{I_0}{I_2} = 1 + KP_2 \tag{17}$$

Dividing equation (17) by equation (16) gives

$$\frac{I_1}{I_2} = \frac{1 + KP_2}{1 + KP_1} \tag{18}$$

This equation may be rearranged to give

$$\frac{I_1}{I_2} = \frac{1}{1 + KP_1} + \frac{K}{1 + KP_1} P_2 \tag{19}$$

The terms $K$ and $P_1$ are constants. The term $P$ may be substituted for $P_2$ because $P_2$ is the variable pressure, and the term $I$, the measured intensity, may be substituted for $I_2$. Thus equation (19) may be expressed as

$$\frac{I_1}{I} = A + BP \tag{20}$$

where

$$A = \frac{1}{1 + KP_1} \tag{21}$$

$$B = \frac{K}{1 + KP_1} \tag{22}$$

Based on the definition of $E$ given in equation (2), $E$ can now be expressed as

$$E = \frac{I_1}{I_0}(A + BP)^{-1} \tag{23}$$

If this equation is treated as the classical Stern-Volmer relation, the results are equivalent and the plots given in figures 1 to 4 have the same shape.

Figure 5 is based on equation (23) and shows a comparison of the emittance and the pressure for two values of $K$ when the wind off luminescence at 13 psia is used as the reference. Figure 5 shows more clearly than figure 3 that the sensitivity in this pressure range is better for the smaller value of $K$ than it is for the larger value of $K$. It should be pointed out that in a plot of $I_1/I$ (as given by eq. (20)), for the two values of $K$ the steeper slope will correspond to the larger value of $K$. This steeper slope should not be interpreted as meaning that the sensitivity is better for the larger value of $K$. The intensity values must be normalized to $I_0$ before comparing sensitivities.

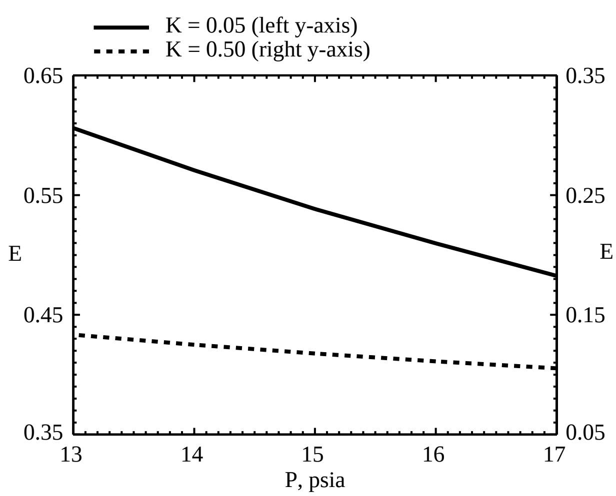


Figure 5. Emittance as a function of pressure for two values of $K$ with a wind off intensity as reference intensity.

## Concluding Remarks

Emittance is a measure of the relative light output of the pressure sensitive paint luminophor; it is not a measure of the absolute emission intensity. The magnitude of emission intensity at oxygen partial pressure of zero is a measure of the output of the luminophor and is related to the quantum efficiency of the luminescence process. One could be working in the optimum range of emittance (0.2–0.8), but have a signal intensity so weak that the noise would cause significant error. In other words, emittance is a relative term and indicates nothing about the signal-to-noise ratio and its effect on relative error. Having a high value of emission intensity at oxygen partial pressure of zero assures a good signal-to-noise ratio.

When considering the optimum pressure range for a paint with a given value for the Stern-Volmer constant, the value for total pressure in the tunnel should correspond to the pressure of minimum error. Model surface pressures are measured as changes in pressure with respect to the total pressure.

In practical applications, optimum experimental conditions should meet the following conditions:

1. Measurements should be made where the total wind tunnel pressure will cause luminescence intensity values corresponding to emittance between 0.2 and 0.8.
2. The Stern-Volmer constant should be as close to the inverse of the total air pressure as possible. In practice it is very difficult to adjust the Stern-Volmer constant. Smaller values of the constant allow a broader working pressure range. However, sensitivity will be lower for smaller values of the constant than for larger values of the constant in their optimum pressure ranges.

NASA Langley Research Center
Hampton, VA 23681-0001
May 24, 1995

## References

1. Morris, M. J.; Donovan, J. F.; Kegelman, J. T.; Schwab, S. D.; Levy, R. L.; and Crites, R. C.: Aerodynamic Application of Pressure-Sensitive Paint. AIAA-92-0264, Jan. 1992.
2. Morris, M. J.; Benne, M. E.; Crites, R. C.; and Donovan, J. F.: Aerodynamic Measurements Based on Photoluminescence. AIAA-93-0175, Jan. 1993.
3. Crites, R. C.: Pressure Sensitive Paint Technique. *VKI, Meas. Tech.*, 1993.
4. Sajben, Miklos: Uncertainty Estimates for Pressure Sensitive Paint Measurements. *AIAA J.*, vol. 31, no. 11, Nov. 1993, pp. 2105–2110.

| REPORT DOCUMENTATION PAGE | | | Form Approved OMB No. 0704-0188 |
|---|---|---|---|



| 1. AGENCY USE ONLY *(Leave blank)* | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | June 1995 | Technical Memorandum |

**4. TITLE AND SUBTITLE**
Optimization of Measurements With Pressure Sensitive Paints

**5. FUNDING NUMBERS**
WU 505-59-54-02

**6. AUTHOR(S)**
Donald M. Oglesby, Chith K. Puram, and Billy T. Upchurch

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
NASA Langley Research Center
Hampton, VA 23681-0001

**8. PERFORMING ORGANIZATION REPORT NUMBER**
L-17474

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
National Aeronautics and Space Administration
Washington, DC 20546-0001

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
NASA TM-4695

**11. SUPPLEMENTARY NOTES**
Oglesby: Old Dominion University, Norfolk, VA; Puram: ViGYAN, Inc., Hampton, VA; Upchurch: Langley Research Center, Hampton, VA.

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**
Unclassified–Unlimited
Subject Category 35
Availability: NASA CASI (301) 621-0390

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*
Use of luminescent paints for the measurement of global pressures on wind tunnel model surfaces requires a full understanding of the inherent accuracy of the technique. Theoretical emission of the paint luminophor follows the well known Stern-Volmer relation. Inherent in this relation are fundamental limits to achievable sensitivity and accuracy. Equations for relative error in pressure as a function of relative signal intensity (emittance), relative error in pressure as a function of pressure, and the relationship between sensitivity and pressure are derived and represented graphically.

**14. SUBJECT TERMS**
Pressure measurement; Pressure sensitive paint; Pressure error; Fluorescence; Phosphorescence; Stern-Volmer equation

**15. NUMBER OF PAGES**
6

**16. PRICE CODE**
A02

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | |